

Flow-shop problems with a single server

- maximal polynomially solvable:

$F2 C_{max}$	Johnson (1954) [3]
$F2; S1 p_{ij} = p; s_{ij} = s; r_i C_{max}$	Brucker et al. (2005) [1]
$F; S1 p_{ij} = 1; s_{ij} = s; r_i C_{max}$	Brucker et al. (2005) [1]
$F; S1 p_{ij} = p; s_{ij} = s C_{max}$	Brucker et al. (2005) [1]
$F2; S1 p_{ij} = p; s_{ij} = s; r_i \sum C_i$	Brucker et al. (2005) [1]
$F; S1 p_{ij} = 1; s_{ij} = s; r_i \sum C_i$	Brucker et al. (2005) [1]
$F p_{ij} = p; r_i \sum w_i C_i$	Single-machine problem
$F2; S1 p_{ij} = p; s_{ij} = 1; r_i \sum w_i C_i$	Single-machine problem
$F p_{ij} = p; r_i \sum w_i U_i$	Single-machine problem
$F2; S1 p_{ij} = p; s_{ij} = 1; r_i \sum w_i U_i$	Single-machine problem
$F2; S1 p_{ij} = p; s_{ij} = s \sum w_i U_i$	Assignment problem
$F; S1 p_{ij} = 1; s_{ij} = s \sum w_i U_i$	Assignment problem
$F p_{ij} = p; r_i \sum T_i$	Single-machine problem
$F2; S1 p_{ij} = p; s_{ij} = 1; r_i \sum T_i$	Single-machine problem
$F p_{ij} = 1; r_i \sum w_i T_i$	Single-machine problem
$F p_{ij} = p \sum w_i T_i$	Single-machine problem
$F2; S1 p_{ij} = p; s_{ij} = s \sum w_i T_i$	Assignment problem
$F; S1 p_{ij} = 1; s_{ij} = s \sum w_i T_i$	Assignment problem

- minimal NP-hard:

$F2; S1 p_{ij} = p C_{max}$	Brucker et al. (2005) [1]
* $F2 r_i C_{max}$	Lenstra et al. (1977) [4]
* $F2; S1 s_{ij} = s C_{max}$	Brucker et al. (2005) [1]
* $F3 C_{max}$	Garey et al. (1976) [2]
* $F2 L_{max}$	Lenstra et al. (1977) [4]
* $F2; S1 p_{ij} = 1; r_i L_{max}$	Brucker et al. (2005) [1]
* $F3; S1 p_{ij} = 1; r_i L_{max}$	Brucker et al. (2005) [1]
* $F2 \sum C_i$	Garey et al. (1976) [2]
* $F2; S1 p_{ij} = 1; r_i \sum C_i$	Brucker et al. (2005) [1]
* $F3; S1 p_{ij} = 1; r_i \sum C_i$	Brucker et al. (2005) [1]
$F2; S1 p_{ij} = 1 \sum w_i U_i$	Brucker et al. (2005) [1]
$F3; S1 p_{ij} = 1 \sum w_i U_i$	Brucker et al. (2005) [1]
$F2; S1 p_{ij} = 1 \sum T_i$	Brucker et al. (2005) [1]
$F3; S1 p_{ij} = 1 \sum T_i$	Brucker et al. (2005) [1]
* $F2; S1 p_{ij} = 1 \sum w_i T_i$	Brucker et al. (2005) [1]
* $F3; S1 p_{ij} = 1 \sum w_i T_i$	Brucker et al. (2005) [1]

- minimal open:

$F2; S1 p_{ij} = 1 C_{max}$	$F3; S1 p_{ij} = 1; s_{ij} = 1; r_i L_{max}$	$F2; S1 p_{ij} = 1; s_{ij} = s; r_i \sum w_i C_i$
$F2; S1 s_{ij} = 1 C_{max}$	$F3; S1 p_{ij} = p; s_{ij} = 1 L_{max}$	$F3; S1 p_{ij} = 1; s_{ij} = 1; r_i \sum w_i C_i$
$F3; S1 p_{ij} = 1 C_{max}$	$F2; S1 p_{ij} = 1 \sum C_i$	$F2 p_{ij} = p; r_i \sum w_i T_i$
$F3; S1 p_{ij} = p; s_{ij} = 1; r_i C_{max}$	$F3; S1 p_{ij} = 1 \sum C_i$	$F2; S1 p_{ij} = 1; s_{ij} = 1; r_i \sum w_i T_i$
$F2; S1 p_{ij} = 1; s_{ij} = s; r_i L_{max}$	$F3; S1 p_{ij} = p; s_{ij} = 1 \sum C_i$	$F3 p_{ij} = p; r_i \sum w_i T_i$

- maximal open:

$F2; S1 s_{ij} = 1 C_{max}$	$F3; S1 p_{ij} = p \sum U_i$
$F3; S1 p_{ij} = p; r_i C_{max}$	$F; S1 p_{ij} = 1 \sum U_i$
$F; S1 p_{ij} = 1; r_i C_{max}$	$F; S1 p_{ij} = p; s_{ij} = s; r_i \sum w_i U_i$
$F; S1 p_{ij} = p \sum w_i C_i$	$F; S1 p_{ij} = p; s_{ij} = s; r_i \sum w_i T_i$

References

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- [2] M.R. Garey, D.S. Johnson, and R. Sethi. The complexity of flowshop and jobshop scheduling. *Math. Oper. Res.*, 1(2):117–129, 1976.
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