

Flow-shop problems without preemption

- **maximal polynomially solvable:**

$F p_{ij} = 1; outtree; r_i C_{max}$	Bruno et al. (1980) [4]
$F p_{ij} = 1; tree C_{max}$	Bruno et al. (1980) [4]
$F2 C_{max}$	Johnson (1954) [6]
$F p_{ij} = 1; intree L_{max}$	Bruno et al. (1980) [4]
$F2 p_{ij} = 1; prec; r_i L_{max}$	Bruno et al. (1980) [4]
$F p_{ij} = 1; outtree; r_i \sum C_i$	Brucker & Knust (1999) [3]
$F2 p_{ij} = 1; prec; r_i \sum C_i$	Baptiste & Timkovsky (2004) [2]
$Fm p_{ij} = 1; intree \sum C_i$	Averbakh et al. (2005) [1]
$F p_{ij} = 1; r_i \sum w_i U_i$	Single-machine problem
$F p_{ij} = 1; r_i \sum w_i T_i$	Single-machine problem

- **minimal NP-hard:**

* $F p_{ij} = 1; intree; r_i C_{max}$	Brucker & Knust (1999) [3]
* $F p_{ij} = 1; prec C_{max}$	Leung et al. (1984) [8], Timkovsky (2003) [10]
* $F2 chains C_{max}$	Lenstra et al. (1977) [7]
* $F2 r_i C_{max}$	Lenstra et al. (1977) [7]
* $F3 C_{max}$	Garey et al. (1976) [5]
* $F p_{ij} = 1; outtree L_{max}$	Brucker & Knust (1999) [3]
* $F2 L_{max}$	Lenstra et al. (1977) [7]
* $F2 \sum C_i$	Garey et al. (1976) [5]
* $F2 p_{ij} = 1; chains \sum w_i C_i$	Tanaev et al. (1994) [9]
* $F3 p_{ij} = 1; chains \sum w_i C_i$	Tanaev et al. (1994) [9]
* $F2 p_{ij} = 1; chains \sum U_i$	Brucker & Knust (1999) [3]
* $F3 p_{ij} = 1; chains \sum U_i$	Brucker & Knust (1999) [3]
* $F2 p_{ij} = 1; chains \sum T_i$	Brucker & Knust (1999) [3]
* $F3 p_{ij} = 1; chains \sum T_i$	Brucker & Knust (1999) [3]

- **minimal open:**

$F3 p_{ij} = 1; intree; r_i C_{max}$	$F3 p_{ij} = 1; chains; r_i L_{max}$	$F p_{ij} = 1; intree \sum C_i$
$F3 p_{ij} = 1; prec C_{max}$	$F3 p_{ij} = 1; outtree L_{max}$	$F3 p_{ij} = 1; intree; r_i \sum C_i$
		$F3 p_{ij} = 1; tree \sum C_i$

- **maximal open:**

$F p_{ij} = 1; chains; r_i L_{max}$	$Fm p_{ij} = 1; prec; r_i L_{max}$	$F p_{ij} = 1; prec; r_i \sum C_i$
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References

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