Shop problems with multipurpose machines

- **maximal polymomially solvable:**

$FMPMm \mid r_i; p_{ij}=1 \mid C_{max}$	Brucker et al. (1997) [4]
$FMPMm \mid r_i; p_{ij}=1 \mid \sum C_i$	Brucker et al. (1997) [4]
$FMPMm \mid p_{ij}=1 \mid \sum w_iC_i$	Brucker et al. (1997) [4]
$FMPMm \mid p_{ij}=1 \mid \sum w_iU_i$	Brucker et al. (1997) [4]
$FMPMm \mid prec; r_i; n=k \mid \sum w_iU_i$	Brucker (1995) [2]
$FMPMm \mid p_{ij}=1 \mid \sum T_i$	Brucker et al. (1997) [4]
$FMPMm \mid prec; r_i; n=k \mid \sum w_iT_i$	Brucker (1995) [2]
$JMPM \mid prec; r_i; n=2 \mid L_{max}$	Jurisch (1995) [7]
$JMPM \mid prec; r_i; p_{ij}=1; n=k \mid \sum w_iU_i$	Brucker et al. (1997) [4]
$JMPM \mid prec; r_i; p_{ij}=1; n=k \mid \sum w_iT_i$	Brucker et al. (1997) [4]
$OMPMm \mid r_i; p_{ij}=1 \mid C_{max}$	Brucker et al. (1997) [4]
$OMPMm \mid r_i; p_{ij}=1 \mid \sum C_i$	Brucker et al. (1997) [4]
$OMPMm \mid p_{ij}=1 \mid \sum w_iC_i$	Brucker et al. (1997) [4]
$OMPMm \mid p_{ij}=1 \mid \sum w_iU_i$	Brucker et al. (1997) [4]
$OMPMm \mid prec; r_i; n=k \mid \sum w_iU_i$	Brucker (1995) [2]
$OMPMm \mid p_{ij}=1 \mid \sum T_i$	Brucker et al. (1997) [4]
$OMPMm \mid prec; r_i; n=k \mid \sum w_iT_i$	Brucker (1995) [2]

- **maximal pseudopolymomially solvable:**

$JMPM2 \mid n=3 \mid C_{max}$	Jurisch (1995) [7], Meyer (1992) [16]
$OMPM \mid p_{ij}=1 \mid C_{max}$	Jurisch (1995) [7]

- **minimal NP-hard:**

	$FMPM \mid n=3 \mid C_{max}$	Brucker et al. (1997) [4]
	$FMPM2 \mid\mid C_{max}$	Lenstra et al. (1977) [15]
*	$FMPM \mid intree; r_i; p_{ij}=1 \mid C_{max}$	Brucker et al. (1977) [3]
*	$FMPM \mid prec; p_{ij}=1 \mid C_{max}$	Ullman (1975) [22]
*	$FMPM2 \mid chains \mid C_{max}$	Lenstra et al. (1977) [15]
*	$FMPM2 \mid r_i \mid C_{max}$	Lenstra et al. (1977) [15]
*	$FMPM3 \mid\mid C_{max}$	Garey et al. (1976) [5]
*	$FMPM \mid outtree; p_{ij}=1 \mid L_{max}$	Brucker et al. (1977) [3]
*	$FMPM2 \mid\mid L_{max}$	Lenstra et al. (1977) [15]
*	$FMPM \mid prec; p_{ij}=1 \mid \sum C_i$	Lenstra & Rinnooy Kan (1978) [13]
*	$FMPM2 \mid\mid \sum C_i$	Garey et al. (1976) [5]
*	$FMPM2 \mid chains; p_{ij}=1 \mid \sum w_iC_i$	Tanaev et al. (1994) [18]
*	$FMPM2 \mid chains; p_{ij}=1 \mid \sum U_i$	Single-machine problem
*	$FMPM2 \mid chains; p_{ij}=1 \mid \sum T_i$	Single-machine problem
	$JMPM2 \mid n=3 \mid C_{max}$	Jurisch (1995) [7], Meyer (1992) [16]
*	$JMPM2 \mid chains; p_{ij}=1 \mid C_{max}$	Timkovsky (1985) [19]
*	$JMPM3 \mid p_{ij}=1 \mid C_{max}$	Lenstra & Rinnooy Kan (1979) [14]
	$JMPM2 \mid r_i; p_{ij}=1 \mid \sum C_i$	Timkovsky (1998) [20]
	$JMPM3 \mid n=3 \mid \sum C_i$	Sotskov & Shakhlevich (1995) [17]
*	$JMPM2 \mid chains; p_{ij}=1 \mid \sum C_i$	Timkovsky (1998) [20]
*	$JMPM3 \mid p_{ij}=1 \mid \sum C_i$	Lenstra (-) [12]
	$JMPM2 \mid p_{ij}=1 \mid \sum w_iC_i$	Timkovsky (1998) [20]
*	$JMPM2 \mid r_i; p_{ij}=1 \mid \sum w_iC_i$	Timkovsky (1998) [20]
	$JMPM2 \mid r_i; p_{ij}=1 \mid \sum U_i$	Timkovsky (1998) [20]
	$JMPM2 \mid p_{ij}=1 \mid \sum w_iU_i$	Kravchenko (1999A) [8]
*	$JMPM2 \mid p_{ij}=1 \mid \sum w_iT_i$	Timkovsky (1998) [20]

	$OMPM \vert n=3 \vert C_{max}$	Gonzalez & Sahni (1976) [6]
	$OMPM3 \vert\vert C_{max}$	Gonzalez & Sahni (1976) [6]
*	$OMPM \vert\vert C_{max}$	Lenstra (-) [12]
*	$OMPM \vert outtree; r_i; p_{ij}=1 \vert C_{max}$	Timkovsky (2003) [21]
*	$OMPM \vert prec; p_{ij}=1 \vert C_{max}$	Timkovsky (2003) [21]
*	$OMPM2 \vert chains \vert C_{max}$	Tanaev et al. (1994) [18]
*	$OMPM2 \vert r_i \vert C_{max}$	Lawler et al. (1981,1982) [10] [11]
*	$OMPM \vert outtree; p_{ij}=1 \vert L_{max}$	Timkovsky (2003) [21]
*	$OMPM2 \vert\vert L_{max}$	Lawler et al. (1981,1982) [10] [11]
*	$OMPM2 \vert\vert \sum C_i$	Achugbue & Chin (1982) [1]
*	$OMPM \vert prec; p_{ij}=1 \vert \sum C_i$	Lenstra & Rinnooy Kan (1978) [13]
*	$OMPM2 \vert chains; p_{ij}=1 \vert \sum w_i C_i$	Timkovsky (2003) [21]
*	$OMPM \vert r_i; p_{ij}=1 \vert \sum U_i$	Kravchenko (1999) [9]
*	$OMPM2 \vert chains; p_{ij}=1 \vert \sum U_i$	Timkovsky (2003) [21]
*	$OMPM2 \vert chains; p_{ij}=1 \vert \sum T_i$	Timkovsky (2003) [21]

- **minimal open:**

$FMPM \vert p_{ij}=1 \vert C_{max}$		
$FMPM2 \vert chains; p_{ij}=1 \vert C_{max}$	$JMPM2 \vert p_{ij}=1 \vert C_{max}$	$OMPM2 \vert chains; p_{ij}=1 \vert C_{max}$
$FMPM2 \vert r_i; p_{ij}=1 \vert L_{max}$	$JMPM2 \vert n=2 \vert \sum C_i$	$OMPM2 \vert r_i; p_{ij}=1 \vert L_{max}$
$FMPM \vert n=2 \vert \sum C_i$	$JMPM2 \vert p_{ij}=1 \vert \sum C_i$	$OMPM \vert p_{ij}=1; n=2 \vert \sum C_i$
$FMPM \vert p_{ij}=1 \vert \sum C_i$	$JMPM2 \vert n=2 \vert \sum U_i$	$OMPM \vert p_{ij}=1; n=3 \vert \sum C_i$
$FMPM2 \vert chains; p_{ij}=1 \vert \sum C_i$	$OMPM \vert p_{ij}=1; n=2 \vert C_{max}$	$OMPM2 \vert chains; p_{ij}=1 \vert \sum C_i$
$FMPM2 \vert r_i; p_{ij}=1 \vert \sum w_i C_i$	$OMPM \vert p_{ij}=1; n=3 \vert C_{max}$	$OMPM2 \vert r_i; p_{ij}=1 \vert \sum w_i C_i$
$FMPM \vert n=2 \vert \sum U_i$	$OMPM2 \vert\vert C_{max}$	$OMPM2 \vert p_{ij}=1 \vert \sum w_i T_i$
$FMPM2 \vert p_{ij}=1 \vert \sum w_i T_i$		

- **maximal open:**

$FMPM \vert outtree; r_i; p_{ij}=1 \vert C_{max}$	
$FMPM \vert tree; p_{ij}=1 \vert C_{max}$	$OMPM \vert tree; p_{ij}=1 \vert C_{max}$
$FMPM \vert chains; r_i; p_{ij}=1 \vert L_{max}$	$OMPM2 \vert\vert C_{max}$
$FMPM \vert intree; p_{ij}=1 \vert L_{max}$	$OMPM \vert intree; r_i; p_{ij}=1 \vert L_{max}$
$FMPMm \vert prec; r_i; p_{ij}=1 \vert L_{max}$	$OMPMm \vert prec; r_i; p_{ij}=1 \vert L_{max}$
$FMPM \vert tree; r_i; p_{ij}=1 \vert \sum C_i$	$OMPM \vert tree; r_i; p_{ij}=1 \vert \sum C_i$
$FMPMm \vert prec; r_i; p_{ij}=1 \vert \sum C_i$	$OMPMm \vert prec; r_i; p_{ij}=1 \vert \sum C_i$
$FMPM \vert prec; r_i; n=k \vert \sum w_i C_i$	$OMPM \vert prec; r_i; n=k \vert \sum w_i C_i$
$FMPM \vert r_i; p_{ij}=1 \vert \sum w_i U_i$	$OMPM \vert p_{ij}=1 \vert \sum w_i U_i$
$FMPM \vert r_i; p_{ij}=1 \vert \sum w_i T_i$	$OMPM \vert prec; r_i; n=2 \vert \sum w_i U_i$
$JMPM2 \vert r_i; p_{ij}=1 \vert L_{max}$	$OMPM \vert prec; r_i; p_{ij}=1; n=k \vert \sum w_i U_i$
$JMPM2 \vert prec; r_i; n=k \vert \sum w_i C_i$	$OMPMm \vert r_i; p_{ij}=1 \vert \sum w_i U_i$
$JMPM2 \vert p_{ij}=1 \vert \sum U_i$	$OMPM \vert prec; r_i; n=2 \vert \sum w_i T_i$
$JMPM \vert prec; r_i; n=2 \vert \sum w_i U_i$	$OMPM \vert prec; r_i; p_{ij}=1; n=k \vert \sum w_i T_i$
$JMPM2 \vert p_{ij}=1 \vert \sum T_i$	$OMPM \vert r_i; p_{ij}=1 \vert \sum w_i T_i$
$JMPM \vert prec; r_i; n=2 \vert \sum w_i T_i$	

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