Shop problems with multiprocessor tasks

- **maximal polymomially solvable:**

$FMPTm \mid r_i; p_{ij} = 1 \mid C_{max}$	Brucker & Kraemer (1996) [5]
$FMPTm \mid r_i; p_{ij} = 1 \mid \sum C_i$	Brucker & Kraemer (1996) [5]
$FMPTm \mid p_{ij} = 1 \mid \sum w_i C_i$	Brucker & Kraemer (1996) [5]
$FMPTm \mid p_{ij} = 1 \mid \sum w_i U_i$	Brucker & Kraemer (1996) [5]
$FMPTm \mid prec; r_i; n = k \mid \sum w_i U_i$	Kraemer (1995) [8]
$FMPTm \mid p_{ij} = 1 \mid \sum T_i$	Brucker & Kraemer (1996) [5]
$FMPTm \mid prec; r_i; n = k \mid \sum w_i T_i$	Kraemer (1995) [8]
$JMPT2 \mid n = k \mid C_{max}$	Brucker & Kraemer (1995) [4]
$JMPT \mid n = 2 \mid \sum w_i U_i$	Brucker (1995) [2]
$JMPT \mid prec; r_i; p_{ij} = 1; n = k \mid \sum w_i U_i$	Brucker & Kraemer (1996) [5]
$JMPT \mid n = 2 \mid \sum w_i T_i$	Brucker (1995) [2]
$JMPT \mid prec; r_i; p_{ij} = 1; n = k \mid \sum w_i T_i$	Brucker & Kraemer (1996) [5]
$OMPT2 \mid\mid C_{max}$	Brucker & Kraemer (1995) [4]
$OMPTm \mid r_i; p_{ij} = 1 \mid C_{max}$	Brucker & Kraemer (1996) [5]
$OMPTm \mid r_i; p_{ij} = 1 \mid \sum C_i$	Brucker & Kraemer (1996) [5]
$OMPTm \mid p_{ij} = 1 \mid \sum w_i C_i$	Brucker & Kraemer (1996) [5]
$OMPT \mid prec; r_i; p_{ij} = 1; n = 2 \mid \sum w_i U_i$	Kraemer (1995) [8]
$OMPTm \mid p_{ij} = 1 \mid \sum w_i U_i$	Brucker & Kraemer (1996) [5]
$OMPTm \mid prec; r_i; n = k \mid \sum w_i U_i$	Kraemer (1995) [8]
$OMPTm \mid p_{ij} = 1 \mid \sum T_i$	Brucker & Kraemer (1996) [5]
$OMPT \mid prec; r_i; p_{ij} = 1; n = 2 \mid \sum w_i T_i$	Kraemer (1995) [8]
$OMPTm \mid prec; r_i; n = k \mid \sum w_i T_i$	Kraemer (1995) [8]

- **minimal NP-hard:**

	$FMPT \mid n = 3 \mid C_{max}$	Kraemer (1995) [8]
*	$FMPT \mid prec; p_{ij} = 1 \mid C_{max}$	Timkovsky (2003) [16]
*	$FMPT \mid tree; r_i; p_{ij} = 1 \mid C_{max}$	Brucker & Knust (1999) [3]
*	$FMPT2 \mid\mid C_{max}$	Brucker & Kraemer (1995) [4]
*	$FMPT \mid tree; p_{ij} = 1 \mid L_{max}$	Brucker & Knust (1999) [3]
*	$FMPT2 \mid\mid \sum C_i$	Garey et al. (1976) [6]
*	$FMPT2 \mid chains; p_{ij} = 1 \mid \sum w_i C_i$	Tanaev et al. (1994) [14]
*	$FMPT2 \mid chains; p_{ij} = 1 \mid \sum U_i$	Single-machine problem
*	$FMPT2 \mid chains; p_{ij} = 1 \mid \sum T_i$	Single-machine problem
	$JMPT3 \mid n = 3 \mid C_{max}$	Sotskov & Shakhlevich (1995) [13]
*	$JMPT2 \mid p_{ij} = 1 \mid C_{max}$	Brucker & Kraemer (1995) [4]
	$JMPT2 \mid r_i; p_{ij} = 1 \mid \sum C_i$	Timkovsky (1998) [15]
	$JMPT3 \mid n = 3 \mid \sum C_i$	Sotskov & Shakhlevich (1995) [13]
*	$JMPT2 \mid chains; p_{ij} = 1 \mid \sum C_i$	Timkovsky (1998) [15]
*	$JMPT3 \mid p_{ij} = 1 \mid \sum C_i$	Lenstra (-) [12]
	$JMPT2 \mid p_{ij} = 1 \mid \sum w_i C_i$	Timkovsky (1998) [15]
*	$JMPT2 \mid r_i; p_{ij} = 1 \mid \sum w_i C_i$	Timkovsky (1998) [15]
	$OMPT \mid n = 3 \mid C_{max}$	Gonzalez & Sahni (1976) [7]
	$OMPT3 \mid\mid C_{max}$	Gonzalez & Sahni (1976) [7]
*	$OMPT \mid\mid C_{max}$	Lenstra (-) [12]
*	$OMPT \mid prec; p_{ij} = 1 \mid C_{max}$	Timkovsky (2003) [16]
*	$OMPT \mid tree; r_i; p_{ij} = 1 \mid C_{max}$	Timkovsky (2003) [16]
*	$OMPT2 \mid chains \mid C_{max}$	Tanaev et al. (1994) [14]
*	$OMPT2 \mid r_i \mid C_{max}$	Lawler et al. (1981,1982) [10] [11]
*	$OMPT \mid tree; p_{ij} = 1 \mid L_{max}$	Timkovsky (2003) [16]
*	$OMPT2 \mid\mid L_{max}$	Lawler et al. (1981,1982) [10] [11]
*	$OMPT2 \mid\mid \sum C_i$	Achugbue & Chin (1982) [1]
*	$OMPT2 \mid chains; p_{ij} = 1 \mid \sum w_i C_i$	Timkovsky (2003) [16]
*	$OMPT \mid r_i; p_{ij} = 1 \mid \sum U_i$	Kravchenko (1999) [9]
*	$OMPT2 \mid chains; p_{ij} = 1 \mid \sum U_i$	Timkovsky (2003) [16]
*	$OMPT2 \mid chains; p_{ij} = 1 \mid \sum T_i$	Timkovsky (2003) [16]

- **minimal open:**

$FMPT\|chains;n=2\|C_{max}$	
$FMPT\|p_{ij}=1\|C_{max}$	$JMPT2\|chains;n=2\|\sum C_i$
$FMPT\|r_i;n=2\|C_{max}$	$JMPT2\|n=3\|\sum C_i$
$FMPT2\|chains;p_{ij}=1\|C_{max}$	$JMPT2\|p_{ij}=1\|\sum C_i$
$FMPT2\|r_i;p_{ij}=1\|L_{max}$	$JMPT2\|r_i;n=2\|\sum C_i$
$FMPT\|chains;n=2\|\sum C_i$	$OMPT\|n=2\|C_{max}$
$FMPT\|n=3\|\sum C_i$	$OMPT\|p_{ij}=1;n=3\|C_{max}$
$FMPT\|p_{ij}=1\|\sum C_i$	$OMPT2\|chains;p_{ij}=1\|C_{max}$
$FMPT\|r_i;n=2\|\sum C_i$	$OMPT2\|r_i;p_{ij}=1\|L_{max}$
$FMPT2\|chains;p_{ij}=1\|\sum C_i$	$OMPT\|n=2\|\sum C_i$
$FMPT2\|r_i;p_{ij}=1\|\sum w_iC_i$	$OMPT\|p_{ij}=1;n=3\|\sum C_i$
$FMPT2\|p_{ij}=1\|\sum w_iT_i$	$OMPT2\|chains;p_{ij}=1\|\sum C_i$
$JMPT2\|chains;n=2\|C_{max}$	$OMPT2\|r_i;p_{ij}=1\|\sum w_iC_i$
$JMPT2\|r_i;n=2\|C_{max}$	$OMPT2\|p_{ij}=1\|\sum w_iT_i$
$JMPT2\|n=3\|L_{max}$	

- **maximal open:**

$FMPT\|tree;p_{ij}=1\|C_{max}$	$OMPT\|tree;p_{ij}=1\|C_{max}$
$FMPT\|chains;r_i;p_{ij}=1\|L_{max}$	$OMPT\|chains;r_i;p_{ij}=1\|L_{max}$
$FMPTm\|prec;r_i;p_{ij}=1\|L_{max}$	$OMPTm\|prec;r_i;p_{ij}=1\|L_{max}$
$FMPT\|prec;r_i;p_{ij}=1\|\sum C_i$	$OMPT\|prec;r_i;p_{ij}=1\|\sum C_i$
$FMPT\|prec;r_i;n=k\|\sum w_iC_i$	$OMPT\|prec;r_i;n=k\|\sum w_iC_i$
$FMPT\|r_i;p_{ij}=1\|\sum w_iU_i$	$OMPT\|p_{ij}=1\|\sum w_iU_i$
$FMPT\|r_i;p_{ij}=1\|\sum w_iT_i$	$OMPT\|prec;r_i;n=2\|\sum w_iU_i$
$JMPT2\|p_{ij}=1\|\sum C_i$	$OMPT\|prec;r_i;p_{ij}=1;n=k\|\sum w_iU_i$
$JMPT\|prec;r_i;n=2\|\sum w_iU_i$	$OMPTm\|r_i;p_{ij}=1\|\sum w_iU_i$
$JMPT2\|prec;r_i;n=k\|\sum w_iU_i$	$OMPT\|prec;r_i;n=2\|\sum w_iT_i$
$JMPT\|prec;r_i;n=2\|\sum w_iT_i$	$OMPT\|prec;r_i;p_{ij}=1;n=k\|\sum w_iT_i$
$JMPT2\|prec;r_i;n=k\|\sum w_iT_i$	$OMPT\|r_i;p_{ij}=1\|\sum w_iT_i$

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