

Open-shop problems with no wait

• maximal polynomially solvable:

$O p_{ij} = 1; tree; no - wait C_{max}$	Brucker et al. (1993) [3], Hu (1961) [5]
$O2 p_{ij} = 1; chains; r_i; no - wait C_{max}$	Timkovsky (2003) [10]
$O2 p_{ij} = 1; prec; no - wait C_{max}$	Brucker et al. (1993) [3], Coffman & Graham (1972) [4]
$O p_{ij} = 1;intree;no - wait L_{max}$	Brucker et al. (1993) [3], Brucker et al. (1977) [2]
$O p_{ij} = 1; r_i; no - wait L_{max}$	Brucker et al. (1993) [3], Simons (1983) [9]
$O p_{ij} = 1; outtree; no - wait \sum C_i$	Brucker et al. (1993) [3], Hu (1961) [5]
$O p_{ij} = 1; r_i; no - wait \sum C_i$	Timkovsky (2003) [10]
$O2 p_{ij} = 1; chains; r_i; no - wait \sum C_i$	Timkovsky (2003) [10]
$O2 p_{ij} = 1; prec; no - wait \sum C_i$	Timkovsky (2003) [10]
$Om p_{ij} = 1;intree;no - wait \sum C_i$	Baptiste et al. (2004) [1]
$Om p_{ij} = 1; r_i; no - wait \sum w_i C_i$	Timkovsky (2003) [10]
$O p_{ij} = 1; no - wait \sum w_i U_i$	Kubiak et al. (1991) [6]
$Om p_{ij} = 1; r_i; no - wait \sum w_i U_i$	Baptiste et al. (2004) [1]
$Om p_{ij} = 1; r_i; no - wait \sum T_i$	Timkovsky (2003) [10]
$O p_{ij} = 1; no - wait \sum w_i T_i$	Brucker et al. (1993) [3]

• minimal NP-hard:

* $O p_{ij} = 1; prec; no - wait C_{max}$	Ullman (1975) [11]
* $O2 no - wait C_{max}$	Sahni & Cho (1979) [8]
* $O p_{ij} = 1; outtree; no - wait L_{max}$	Brucker et al. (1977) [2]
* $O p_{ij} = 1; prec; no - wait \sum C_i$	Lenstra & Rinnooy Kan (1978) [7]
* $O2 no - wait \sum C_i$	Kubiak et al. (1991) [6]
* $O2 p_{ij} = 1; chains; no - wait \sum w_i C_i$	Timkovsky (2003) [10]
* $O3 p_{ij} = 1; chains; no - wait \sum w_i C_i$	Timkovsky (2003) [10]
* $O2 p_{ij} = 1; chains; no - wait \sum U_i$	Timkovsky (2003) [10]
* $O3 p_{ij} = 1; chains; no - wait \sum U_i$	Timkovsky (2003) [10]
* $O2 p_{ij} = 1; chains; no - wait \sum T_i$	Timkovsky (2003) [10]
* $O3 p_{ij} = 1; chains; no - wait \sum T_i$	Timkovsky (2003) [10]

• minimal open:

$O2 p_{ij} = 1;intree;r_i;no - wait C_{max}$	$O3 p_{ij} = 1; chains; r_i; no - wait \sum C_i$
$O2 p_{ij} = 1; outtree; r_i; no - wait C_{max}$	$O3 p_{ij} = 1; tree; no - wait \sum C_i$
$O3 p_{ij} = 1; chains; r_i; no - wait C_{max}$	$O p_{ij} = 1;intree;no - wait \sum C_i$
$O3 p_{ij} = 1; prec; no - wait C_{max}$	$O p_{ij} = 1; r_i; no - wait \sum w_i C_i$
$O2 p_{ij} = 1; chains; r_i; no - wait L_{max}$	$O p_{ij} = 1; r_i; no - wait \sum U_i$
$O2 p_{ij} = 1; outtree; no - wait L_{max}$	$O p_{ij} = 1; r_i; no - wait \sum T_i$
$O3 p_{ij} = 1; outtree; no - wait L_{max}$	$O2 p_{ij} = 1; r_i; no - wait \sum w_i T_i$
$O2 p_{ij} = 1;intree;r_i;no - wait \sum C_i$	$O3 p_{ij} = 1; r_i; no - wait \sum w_i T_i$
$O2 p_{ij} = 1; outtree; r_i; no - wait \sum C_i$	

• maximal open:

$O p_{ij} = 1; tree; r_i; no - wait C_{max}$	$Om p_{ij} = 1; prec; r_i; no - wait \sum C_i$
$O p_{ij} = 1;intree;r_i;no - wait L_{max}$	$O p_{ij} = 1; r_i; no - wait \sum w_i U_i$
$Om p_{ij} = 1; prec; r_i; no - wait L_{max}$	$O p_{ij} = 1; r_i; no - wait \sum w_i T_i$
$O p_{ij} = 1; tree; r_i; no - wait \sum C_i$	

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