Multiprocessor task problems with parallel processors and preemption

- **maximal polynomially solvable:**

$P\|outtree;pmtn;r_i\|C_{max}$	Lawler (1982) [21]
$P\|tree;pmtn\|C_{max}$	Muntz & Coffman (1970) [30], Gonzalez & Johnson (1980) [16]
$Q\|chains;pmtn\|C_{max}$	Horvath et al. (1977) [17]
$P\|intree;pmtn\|L_{max}$	Lawler (1982) [21]
$Pm\|pmtn;r_i;size_i\|L_{max}$	Blazewicz et al. (1996) [4]
$Q2\|prec;pmtn;r_i\|L_{max}$	Lawler (1982) [21]
$R\|pmtn;r_i\|L_{max}$	Lawler & Labetoulle (1978) [23]
$P2\|p_i=p;prec;pmtn\|\sum C_i$	Coffman et al. (2003) [10]
$P2\|p_i=p;outtree;pmtn;r_i\|\sum C_i$	Lushchakova (2006) [28]
$P\|p_i=p;pmtn;r_i\|\sum C_i$	Brucker & Kravchenko (2004) [8]
$P\|p_i=1;outtree;pmtn;r_i\|\sum C_i$	Brucker et al. (2002) [6], Huo & Leung (2005) [18]
$P\|p_i=p;outtree;pmtn\|\sum C_i$	Brucker et al. (2002) [6]
$Q\|pmtn\|\sum C_i$	Labetoulle et al. (1984) [19]
$P\|p_i=p;pmtn\|\sum w_iC_i$	McNaughton (1959) [29]
$Q\|p_i=p;pmtn\|\sum U_i$	Baptiste et al. (2004) [3]
$Qm\|pmtn\|\sum U_i$	Lawler (1979) [20], Lawler & Martel (1989) [25]
$P\|p_i=1;pmtn;r_i\|\sum w_iU_i$	Brucker et al. (2003) [5]
$Pm\|p_i=p;pmtn\|\sum w_iU_i$	Baptiste (2000B) [1]
$P\|p_i=p;pmtn\|\sum T_i$	Baptiste et al. (2004) [3]
$P\|p_i=1;pmtn;r_i\|\sum w_iT_i$	Baptiste (2002) [2]

- **maximal pseudopolynomially solvable:**

$Pm\|pmtn\|\sum w_iC_i$	McNaughton (1959) [29], Lawler et al. (1989) [24]
$Qm\|pmtn\|\sum w_iU_i$	Lawler (1979) [20], Lawler & Martel (1989) [25]

- **minimal NP-hard:**

	$P\|p_i=1;pmtn;size_i\|C_{max}$	Drozdowski (1992) [11]
*	$P\|intree;pmtn;r_i\|C_{max}$	Lenstra (-) [26]
*	$P\|p_i=1;prec;pmtn\|C_{max}$	Ullman (1976) [33]
*	$R2\|chains;pmtn\|C_{max}$	Lenstra (-) [26]
*	$P\|outtree;pmtn\|L_{max}$	Lenstra (-) [26]
	$P2\|pmtn;r_i\|\sum C_i$	Du et al. (1990) [14]
	$P\|pmtn;size_i\|\sum C_i$	Drozdowski & Dell' Olmo (2000) [12]
*	$P\|pmtn;r_i\|\sum C_i$	Brucker & Kravchenko (2004) [8]
*	$P2\|chains;pmtn\|\sum C_i$	Du et al. (1991) [15]
*	$R\|pmtn\|\sum C_i$	Sitters (2001) [31]
	$P2\|pmtn\|\sum w_iC_i$	Bruno et al. (1974) [9]
*	$P\|p_i=p;pmtn;r_i\|\sum w_iC_i$	Leung & Young (1990A) [27]
*	$P\|pmtn\|\sum w_iC_i$	Lenstra (-) [26]
*	$P2\|p_i=1;chains;pmtn\|\sum w_iC_i$	Timkovsky (2003) [32], Du et al. (1991) [15]
*	$P2\|pmtn;r_i\|\sum w_iC_i$	Labetoulle et al. (1984) [19]
	$P\|pmtn\|\sum U_i$	Lawler (1983) [22]
	$P2\|pmtn;r_i\|\sum U_i$	Du et al. (1992) [13]
*	$P2\|p_i=1;chains;pmtn\|\sum U_i$	Baptiste et al. (2004) [3]
*	$R\|pmtn\|\sum U_i$	Sitters (2001) [31]
	$P\|p_i=p;pmtn\|\sum w_iU_i$	Brucker & Kravchenko (1999) [7]
	$P2\|pmtn\|\sum w_iU_i$	Single-machine problem

- **minimal open:**

$P2\|p_i=1;chains;pmtn;size_i\|C_{max}$	$P2\|p_i=1;pmtn;size_i\|\sum C_i$
$Q2\|p_i=p;pmtn;size_i\|C_{max}$	$P2\|p_i=1;pmtn;size_i\|\sum U_i$

- **maximal open:**

$Qm\|prec;pmtn;r_i;size_i\|L_{max}$	$Rm\|pmtn;size_i\|\sum U_i$
$Rm\|pmtn;r_i;size_i\|L_{max}$	$Qm\|p_i=p;pmtn;r_i;size_i\|\sum w_iU_i$
$Q\|p_i=p;prec;pmtn;r_i;size_i\|\sum C_i$	$Qm\|p_i=p;prec;pmtn;r_i;size_i\|\sum T_i$
$P\|p_i=1;pmtn;r_i;size_i\|\sum w_iC_i$	$Rm\|pmtn;size_i\|\sum T_i$
$Q\|p_i=p;pmtn;size_i\|\sum w_iC_i$	$Qm\|p_i=p;pmtn;r_i;size_i\|\sum w_iT_i$

References

[1] P. Baptiste. Preemptive scheduling of identical machines. Technical Report 2000-314, Universite de Technologie de Compiegne, France, 2000.

[2] P. Baptiste. On preemption redundancy. Technical Report, IBM T.J. Watson Research Center, New York, 2002.

[3] P. Baptiste, P. Brucker, S. Knust, and V. Timkovsky. Ten notes on equal-execution-time scheduling. *4OR*, 2:111–127, 2004.

[4] J. Błażewicz, M. Drozdowski, D. de Werra, and J. Węglarz. Deadline scheduling of multiprocessor tasks. *Discrete Appl. Math.*, 65(1-3):81–95, 1996.

[5] P. Brucker, S. Heitmann, and J. Hurink. How useful are preemptive schedules? *Oper. Res. Lett.*, 31(2):129–136, 2003.

[6] P. Brucker, J. Hurink, and S. Knust. A polynomial algorithm for $P|p_j = 1, r_j, outtree| \sum C_j$. *Math. Methods Oper. Res.*, 56(3):407–412, 2002.

[7] P. Brucker and S.A. Kravchenko. Preemption can make parallel machine scheduling problems hard. OSM Reihe P, Heft 211, Universität Osnabrück, Fachbereich Mathematik/Informatik, 1999.

[8] P. Brucker and S.A. Kravchenko. Complexity of mean flow time scheduling problems with release dates. OSM Reihe P, Heft 251, Universität Osnabrück, Fachbereich Mathematik/Informatik, 2004.

[9] J. Bruno, E.G. Coffman, Jr., and R. Sethi. Scheduling independent tasks to reduce mean finishing time. *Comm. ACM*, 17:382–387, 1974.

[10] E.G. Coffman, Jr., J. Sethuraman, and V.G. Timkovsky. Ideal preemptive schedules on two processors. *Acta Informat.*, 39:597–612, 2003.

[11] M. Drozdowski. *Problems and algorithms of multiprocessor tasks scheduling.* PhD thesis, Technical University of Poznan, Department of Computer Science, 1992.

[12] M. Drozdowski and P. Dell' Olmo. Scheduling multiprocessor tasks for mean flow time criterion. *Comput. Oper. Res.*, 27(6):571–585, 2000.

[13] J. Du, J.Y.-T. Leung, and C.S. Wong. Minimizing the number of late jobs with release time constraint. *J. Combin. Math. Combin. Comput.*, 11:97–107, 1992.

[14] J. Du, J.Y.-T. Leung, and G.H. Young. Minimizing mean flow time with release time constraint. *Theoret. Comput. Sci.*, 75(3):347–355, 1990.

[15] J. Du, J.Y.-T. Leung, and G.H. Young. Scheduling chain-structured tasks to minimize makespan and mean flow time. *Inform. and Comput.*, 92(2):219–236, 1991.

[16] T. Gonzalez and D.B. Johnson. A new algorithm for preemptive scheduling of trees. *J. Assoc. Comput. Mach.*, 27(2):287–312, 1980.

[17] E.C. Horvath, S. Lam, and R. Sethi. A level algorithm for preemptive scheduling. *J. Assoc. Comput. Mach.*, 24(1):32–43, 1977.

[18] Y. Huo and J.Y.-T. Leung. Minimizing total completion time for UET tasks with release time and outtree precedence constraints. *Math. Methods Oper. Res.*, 62(2):275–279, 2005.

[19] J. Labetoulle, E.L. Lawler, J.K. Lenstra, and A.H.G. Rinnooy Kan. Preemptive scheduling of uniform machines subject to release dates. In *Progress in combinatorial optimization (Waterloo, Ont., 1982)*, pages 245–261. Academic Press, Toronto, Ont., 1984.

[20] E.L. Lawler. Preemptive scheduling of uniform parallel machines to minimize the weighted number of late jobs. Report BW 105, Centre for Mathematics and Computer Science, Amsterdam, 1979.

[21] E.L. Lawler. Preemptive scheduling of precedence-constrained jobs on parallel machines. In M.A.H. Dempster, J.K. Lenstra, and A.H.G. Rinnooy Kan, editors, *Deterministic and stochastic scheduling, Proceedings of the NATO Advanced Study and Research Institute on Theoretical Approaches to Scheduling Problems held in Durham, July 6–17, 1981*, volume 84 of *NATO Advanced Study Institute Series C: Mathematical and Physical Sciences*, pages 101–123, Dordrecht, 1982. D. Reidel Publishing Co.

[22] E.L. Lawler. Recent results in the theory of machine scheduling. In A. Bachem, M. Groetschel, and B. Korte, editors, *Mathematical programming: the state of the art (Bonn, 1982)*, pages 202–234. Springer, Berlin, 1983.

[23] E.L. Lawler and J. Labetoulle. On preemptive scheduling of unrelated parallel processors by linear programming. *J. Assoc. Comput. Mach.*, 25(4):612–619, 1978.

[24] E.L. Lawler, J.K. Lenstra, A.H.G. Rinnooy Kan, and D.B. Shmoys. *Sequencing and Scheduling: Algorithms and Complexity*, volume 4 of *Operations Research and Managment Science*. CWI, Amsterdam, 1989.

[25] E.L. Lawler and C.U. Martel. Preemptive scheduling of two uniform machines to minimize the number of late jobs. *Oper. Res.*, 37(2):314–318, 1989.

[26] J.K. Lenstra. Not published.

[27] J.Y.-T. Leung and G.H. Young. Preemptive scheduling to minimize mean weighted flow time. *Inform. Process. Lett.*, 34(1):47–50, 1990.

[28] I. Lushchakova. Two machine preemptive scheduling problem with release dates, equal processing times and precedence constraints. *European J. Oper. Res.*, 171(1):107–122, 2006.

[29] R. McNaughton. Scheduling with deadlines and loss functions. *Management Sci.*, 6:1–12, 1959.

[30] R.R. Muntz and E.G. Coffman, Jr. Preemptive scheduling of real-time tasks on multiprocessor systems. *J. Assoc. Comput. Mach.*, 17:324–338, 1970.

[31] R.A. Sitters. Two NP-hardness results for preemptive minsum scheduling of unrelated parallel machines. In *Proc. 8th International IPCO Conference, Lecture Notes in Computer Science*, pages 396–405. Springer, 2001.

[32] V.G. Timkovsky. Identical parallel machines vs. unit-time shops and preemptions vs. chains in scheduling complexity. *European J. Oper. Res.*, 149(2):355–376, 2003.

[33] J.D. Ullman. Complexity of sequencing problems. In J.L. Bruno, E.G. Coffman, Jr., R.L. Graham, W.H. Kohler, R. Sethi, K. Steiglitz, and J.D. Ullman, editors, *Computer and Job/Shop Scheduling Theory*. John Wiley & Sons Inc., New York, 1976.