Single machine problems with non-negative time-lags

- **maximal polynomially solvable:**

$1\|chains(l); p_i = p\|C_{max}$	Munier & Sourd (2003) [21]
$1\|outtree(l); p_i = 1; r_i\|C_{max}$	Bruno et al. (1980) [6]
$1\|prec(1)\|C_{max}$	Finta & Liu (1996) [8]
$1\|prec; r_i\|C_{max}$	Lawler (1973) [10]
$1\|tree(l); p_i = 1\|C_{max}$	Bruno et al. (1980) [6]
$1\|intree(l); p_i = 1\|L_{max}$	Bruno et al. (1980) [6]
$1\|prec\|L_{max}$	Lawler (1973) [10]
$1\|prec(1); p_i = 1; r_i\|L_{max}$	Bruno et al. (1980) [6]
$1\|prec; p_i = p; r_i\|L_{max}$	Simons (1978) [23]
$1\|chains(l); p_i = p\|\sum C_i$	Brucker et al. (2006) [5]
$1\|outtree(l); p_i = 1; r_i\|\sum C_i$	Brucker & Knust (1999) [4]
$1\|prec(1); p_i = 1; r_i\|\sum C_i$	Baptiste & Timkovsky (2004) [3]
$1\|prec; p_i = p; r_i\|\sum C_i$	Simons (1983) [24]
$1\|p_i = p; r_i\|\sum w_iC_i$	Baptiste (2000) [2]
$1\|sp-graph\|\sum w_iC_i$	Lawler (1978) [12]
$1\|\|\sum U_i$	Moore (1968) [20], Maxwell (1970) [19], Sidney (1973) [22]
$1\|p_i = p; r_i\|\sum w_iU_i$	Baptiste (1999) [1]
$1\|p_i = p; r_i\|\sum T_i$	Baptiste (2000) [2]
$1\|p_i = 1; r_i\|\sum w_iT_i$	Assignment-problem
$1\|p_i = p\|\sum w_iT_i$	Assignment-problem

- **maximal pseudopolynomially solvable:**

$1\|\|\sum w_iU_i$	Lawler & Moore (1969) [13], Karp (1972) [9]
$1\|\|\sum T_i$	Lawler (1977) [11], Du & Leung (1990) [7]

- **minimal NP-hard:**

*	$1\|chains(l)\|C_{max}$	Wikum et al. (1994) [27]
*	$1\|chains(l_{ij}); p_i = 1\|C_{max}$	Yu (1996) [28], Yu et al. (2004) [29]
*	$1\|intree(l); p_i = 1; r_i\|C_{max}$	Brucker & Knust (1999) [4]
*	$1\|prec(l); p_i = 1\|C_{max}$	Leung et al. (1984) [17], Timkovsky (2003) [26]
*	$1\|outtree(l); p_i = 1\|L_{max}$	Brucker & Knust (1999) [4]
*	$1\|r_i\|L_{max}$	Lenstra et al. (1977) [16]
*	$1\|chains(l)\|\sum C_i$	Brucker & Knust (1999) [4]
*	$1\|prec\|\sum C_i$	Lawler (1978) [12], Lenstra & Rinnooy Kan (1978) [14]
*	$1\|r_i\|\sum C_i$	Lenstra et al. (1977) [16]
*	$1\|chains(1); p_i = 1\|\sum w_iC_i$	Tanaev et al. (1994) [25]
*	$1\|chains; p_i = 1; r_i\|\sum w_iC_i$	Lenstra & Rinnooy Kan (1980) [15]
*	$1\|prec; p_i = 1\|\sum w_iC_i$	Lawler (1978) [12], Lenstra & Rinnooy Kan (1978) [14]
*	$1\|chains; p_i = 1\|\sum U_i$	Lenstra & Rinnooy Kan (1980) [15]
	$1\|\|\sum w_iU_i$	Lawler & Moore (1969) [13], Karp (1972) [9]
	$1\|\|\sum T_i$	Lawler (1977) [11], Du & Leung (1990) [7]
*	$1\|chains; p_i = 1\|\sum T_i$	Leung & Young (1990) [18]
*	$1\|\|\sum w_iT_i$	Lawler (1977) [11], Lenstra et al. (1977) [16]

- **minimal open:**

$1\|chains(1); p_i = p; r_i\|C_{max}$	$1\|chains(l_{ij}); p_i = 1\|\sum C_i$
$1\|intree(l); p_i = p\|C_{max}$	$1\|intree(1); p_i = 1; r_i\|\sum C_i$
$1\|outtree(l); p_i = p\|C_{max}$	$1\|intree(1); p_i = p\|\sum C_i$
$1\|chains(1); p_i = p\|L_{max}$	$1\|intree(l); p_i = 1\|\sum C_i$
$1\|chains(l); p_i = 1; r_i\|L_{max}$	$1\|outtree(1); p_i = p\|\sum C_i$
$1\|chains(1)\|\sum C_i$	$1\|p_i = p; r_i\|\sum w_iT_i$
$1\|chains(1); p_i = p; r_i\|\sum C_i$	

- **maximal open:**

$1\|outtree(l); p_i = p; r_i\|C_{max}$	$1\|prec(1)\|L_{max}$
$1\|prec(1); r_i\|C_{max}$	$1\|prec(1); p_i = p; r_i\|L_{max}$
$1\|tree(l); p_i = p\|C_{max}$	$1\|prec(l_{ij}); p_i = p; r_i\|\sum C_i$
$1\|chains(l); p_i = p; r_i\|L_{max}$	$1\|tree(1)\|\sum C_i$
$1\|intree(l); p_i = p\|L_{max}$	$1\|p_i = p; r_i\|\sum w_iT_i$

References

[1] P. Baptiste. Polynomial time algorithms for minimizing the weighted number of late jobs on a single machine with equal processing times. *J. Sched.*, 2:245–252, 1999.

[2] P. Baptiste. Scheduling equal-length jobs on identical parallel machines. *Discrete Appl. Math.*, 103(1):21–32, 2000.

[3] P. Baptiste and V. Timkovsky. Shortest path to nonpreemptive schedules of unit-time jobs on two identical parallel machines with minimum total completion time. *Math. Methods Oper. Res.*, 60(1):145–153, 2004.

[4] P. Brucker and S. Knust. Complexity results for single-machine problems with positive finish-start time-lags. *Computing*, 63:299–316, 1999.

[5] P. Brucker, S. Knust, and C. Oguz. Scheduling chains with identical jobs and constant delays on a single machine. *Math. Methods Oper. Res.*, 63(1):63–75, 2006.

[6] J. Bruno, J.W. Jones, III, and K. So. Deterministic scheduling with pipelined processors. *IEEE Trans. Comput.*, 29(4):308–316, 1980.

[7] J. Du and J.Y.-T. Leung. Minimizing total tardiness on one machine is NP-hard. *Math. Oper. Res.*, 15(3):483–495, 1990.

[8] L. Finta and Z. Liu. Single machine scheduling subject to precedence delays. *Discrete Appl. Math.*, 70(3):247–266, 1996.

[9] R.M. Karp. Reducibility among combinatorial problems. In *Complexity of computer computations (Proc. Sympos., IBM Thomas J. Watson Res. Center, Yorktown Heights, N.Y., 1972)*, pages 85–103. Plenum, New York, 1972.

[10] E.L. Lawler. Optimal sequencing of a single machine subject to precedence constraints. *Management Sci.*, 19:544–546, 1973.

[11] E.L. Lawler. A "pseudopolynomial" algorithm for sequencing jobs to minimize total tardiness. *Ann. of Discrete Math.*, 1:331–342, 1977.

[12] E.L. Lawler. Sequencing jobs to minimize total weighted completion time subject to precedence constraints. *Ann. Discrete Math.*, 2:75–90, 1978.

[13] E.L. Lawler and J.M. Moore. A functional equation and its application to resource allocation and sequencing problems. *Management Sci.*, 16:77–84, 1969.

[14] J.K. Lenstra and A.H.G. Rinnooy Kan. Complexity of scheduling under precedence constraints. *Oper. Res.*, 26(1):22–35, 1978.

[15] J.K. Lenstra and A.H.G. Rinnooy Kan. Complexity results for scheduling chains on a single machine. *European J. Oper. Res.*, 4(4):270–275, 1980.

[16] J.K. Lenstra, A.H.G. Rinnooy Kan, and P. Brucker. Complexity of machine scheduling problems. *Ann. of Discrete Math.*, 1:343–362, 1977.

[17] J.Y.-T. Leung, O. Vornberger, and J.D. Witthoff. On some variants of the bandwidth minimization problem. *SIAM J. Comput.*, 13(3):650–667, 1984.

[18] J.Y.-T. Leung and G.H. Young. Minimizing total tardiness on a single machine with precedence constraints. *ORSA J. Comput.*, 2(4):346–352, 1990.

[19] W.L. Maxwell. On sequencing n jobs on one machine to minimize the number of late jobs. *Management Sci.*, 16:295–29, 1970.

[20] J.M. Moore. An n job, one machine sequencing algorithm for minimizing the number of late jobs. *Management Sci.*, 15:102–109, 1968.

[21] A. Munier and F. Sourd. Scheduling chains on a single machine with non-negative time lags. *Math. Methods Oper. Res.*, 57(1):111–123, 2003.

[22] J.B. Sidney. An extension of Moore's due date algorithm. In *Symposium on the Theory of Scheduling and its Applications (North Carolina State Univ., Raleigh, N. C., 1972)*, pages 393–398. Lecture Notes in Economics and Mathematical Systems, Vol. 86, Berlin, 1973. Springer. Incorporating the results of discussion by Hamilton Emmons and John Rau.

[23] B. Simons. A fast algorithm for single processor scheduling. In *19th Annual Symposium on Foundations of Computer Science (Ann Arbor, Mich., 1978)*, pages 246–252. IEEE, Long Beach, Calif., 1978.

[24] B. Simons. Multiprocessor scheduling of unit-time jobs with arbitrary release times and deadlines. *SIAM J. Comput.*, 12(2):294–299, 1983.

[25] V.S. Tanaev, Y.N. Sotskov, and V.A. Strusevich. *Scheduling theory. Multi-stage systems*, volume 285 of *Mathematics and its Applications*. Kluwer Academic Publishers Group, Dordrecht, 1994. Translated and revised from the 1989 Russian original by the authors.

[26] V.G. Timkovsky. Identical parallel machines vs. unit-time shops and preemptions vs. chains in scheduling complexity. *European J. Oper. Res.*, 149(2):355–376, 2003.

[27] E.D. Wikum, D.C. Llewellyn, and G.L. Nemhauser. One-machine generalized precedence constrained scheduling problems. *Oper. Res. Lett.*, 16(2):87–99, 1994.

[28] W. Yu. *The two-machine flow shop problem with delays and the one-machine total tardiness problem.* PhD thesis, Technische Universiteit Eindhoven, Eindhoven, 1996. Dissertation, Technische Universiteit Eindhoven, Eindhoven, 1996.

[29] W. Yu, H. Hoogeveen, and J.K. Lenstra. Minimizing makespan in a two-machine flow shop with delays and unit-time operations is NP-hard. *J. Sched.*, 7(5):333–348, 2004.