Parallel machine problems without preemption

- **maximal polynomially solvable:**

$P\|p_i = p; outtree; r_i\|C_{max}$	Brucker et al. (1977) [3]
$P\|p_i = p; tree\|C_{max}$	Hu (1961) [18], Davida & Linton (1976) [10]
$Q\|p_i = p; r_i\|C_{max}$	Assignment-problem
$Q2\|p_i = p; chains\|C_{max}$	Brucker et al. (1999) [5]
$P\|p_i = 1; chains; r_i\|L_{max}$	Dror et al. (1998) [12], Baptiste et al. (2004) [1]
$P\|p_i = p; intree\|L_{max}$	Brucker et al. (1977) [3], Monma (1982) [25]
$P2\|p_i = p; prec\|L_{max}$	Garey & Johnson (1976) [14]
$P2\|p_i = 1; prec; r_i\|L_{max}$	Garey & Johnson (1977) [15]
$P\|p_i = 1; outtree; r_i\|\sum C_i$	Brucker et al. (2002) [4], Huo & Leung (2005) [19]
$P\|p_i = p; outtree\|\sum C_i$	Hu (1961) [18]
$P2\|p_i = 1; prec; r_i\|\sum C_i$	Baptiste & Timkovsky (2004) [2]
$P2\|p_i = p; prec\|\sum C_i$	Coffman & Graham (1972) [9]
$Pm\|p_i = p; tree\|\sum C_i$	Baptiste et al. (2004) [1]
$Qm\|p_i = p; r_i\|\sum C_i$	Dessouky et al. (1990) [11]
$R\|\|\sum C_i$	Horn (1973) [17], Bruno et al. (1974) [8]
$P\|p_i = p; r_i\|\sum w_iC_i$	Brucker & Kravchenko (2008) [7]
$P\|p_i = 1; r_i\|\sum w_iU_i$	Networkflowproblem
$Pm\|p_i = p; r_i\|\sum w_iU_i$	Baptiste et al. (2004) [1]
$Q\|p_i = p\|\sum w_iU_i$	Assignment-problem
$P\|p_i = p; r_i\|\sum T_i$	Brucker & Kravchenko (2005) [6]
$P\|p_i = 1; r_i\|\sum w_iT_i$	Networkflowproblem
$Q\|p_i = p\|\sum w_iT_i$	Assignment-problem

- **maximal pseudopolynomially solvable:**

$Qm\|r_i\|C_{max}$	Lawler et al. (1989) [21]
$Qm\|\|\sum w_iC_i$	Lawler et al. (1989) [21]
$Qm\|\|\sum w_iU_i$	Lawler et al. (1989) [21]

- **minimal NP-hard:**

	$P2\|\|C_{max}$	Lenstra et al. (1977) [24]
*	$P\|\|C_{max}$	Garey & Johnson (1978) [16]
*	$P\|p_i = 1; intree; r_i\|C_{max}$	Brucker et al. (1977) [3]
*	$P\|p_i = 1; prec\|C_{max}$	Ullman (1975) [27]
*	$P2\|chains\|C_{max}$	Du et al. (1991) [13]
*	$Q\|p_i = p; chains\|C_{max}$	Kubiak (1988) [20]
*	$P\|p_i = 1; outtree\|L_{max}$	Brucker et al. (1977) [3]
*	$P\|p_i = 1; intree; r_i\|\sum C_i$	Lenstra (-) [22]
*	$P\|p_i = 1; prec\|\sum C_i$	Lenstra & Rinnooy Kan (1978) [23]
*	$P2\|chains\|\sum C_i$	Du et al. (1991) [13]
*	$P2\|r_i\|\sum C_i$	Single-machine problem
	$P2\|\|\sum w_iC_i$	Bruno et al. (1974) [8]
*	$P\|\|\sum w_iC_i$	Lenstra (-) [22]
*	$P2\|p_i = 1; chains\|\sum w_iC_i$	Timkovsky (2003) [26]
*	$P2\|p_i = 1; chains\|\sum U_i$	Single-machine problem
*	$P2\|p_i = 1; chains\|\sum T_i$	Single-machine problem

- **minimal open:**

$P2\|p_i = p; intree; r_i\|C_{max}$	$P2\|p_i = p; chains; r_i\|L_{max}$	$Pm\|p_i = 1; prec\|\sum C_i$
$Pm\|p_i = 1; intree; r_i\|C_{max}$	$Pm\|p_i = 1; outtree\|L_{max}$	$Q\|p_i = p; r_i\|\sum C_i$
$Pm\|p_i = 1; prec\|C_{max}$	$Q2\|p_i = p; chains\|L_{max}$	$Q2\|p_i = p; chains\|\sum C_i$
$Q2\|p_i = p; chains; r_i\|C_{max}$	$Q2\|p_i = p; r_i\|L_{max}$	$Q2\|p_i = p; r_i\|\sum w_iC_i$
$Q2\|p_i = p; intree\|C_{max}$	$P\|p_i = 1; intree\|\sum C_i$	$P\|p_i = p; r_i\|\sum U_i$
$Q2\|p_i = p; outtree\|C_{max}$	$P2\|p_i = p; chains; r_i\|\sum C_i$	$P2\|p_i = p; r_i\|\sum w_iT_i$
$Qm\|p_i = p; chains\|C_{max}$	$Pm\|p_i = 1; intree; r_i\|\sum C_i$	

- **maximal open:**

$P\|p_i = p; chains; r_i\|L_{max}$	$Q\|p_i = p; tree\|\sum C_i$	$Q\|p_i = p; r_i\|\sum w_iU_i$
$Qm\|p_i = p; prec; r_i\|L_{max}$	$Qm\|p_i = p; prec; r_i\|\sum C_i$	$Q\|p_i = p; r_i\|\sum w_iT_i$
$Q\|p_i = p; outtree; r_i\|\sum C_i$		

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