

Parallel machine problems with preemption

- **maximal polynomially solvable:**

$P outtree; pmtn; r_i C_{max}$	Lawler (1982) [19]
$P tree; pmtn C_{max}$	Muntz & Coffman (1970) [28], Gonzalez & Johnson (1980) [13]
$Q chains; pmtn C_{max}$	Horvath et al. (1977) [14]
$P intree; pmtn L_{max}$	Lawler (1982) [19]
$Q2 prec; pmtn; r_i L_{max}$	Lawler (1982) [19]
$R pmtn; r_i L_{max}$	Lawler & Labetoulle (1978) [21]
$P2 p_i = p; prec; pmtn \sum C_i$	Coffman et al. (2003) [9]
$P2 p_i = p; outtree; pmtn; r_i \sum C_i$	Lushchakova (2006) [26]
$P p_i = 1; outtree; pmtn; r_i \sum C_i$	Brucker et al. (2002) [5], Huo & Leung (2005) [15]
$P p_i = p; outtree; pmtn \sum C_i$	Brucker et al. (2002) [5]
$Q p_i = p; pmtn; r_i \sum C_i$	Kravchenko & Werner (2009) [16]
$Q pmtn \sum C_i$	Labetoulle et al. (1984) [17]
$P p_i = p; pmtn \sum w_i C_i$	McNaughton (1959) [27]
$Q p_i = p; pmtn \sum U_i$	Baptiste et al. (2004) [3]
$Qm pmtn \sum U_i$	Lawler (1979) [18], Lawler & Martel (1989) [23]
$P p_i = 1; pmtn; r_i \sum w_i U_i$	Brucker et al. (2003) [4]
$Pm p_i = p; pmtn \sum w_i U_i$	Baptiste (2000B) [1]
$P p_i = p; pmtn \sum T_i$	Baptiste et al. (2004) [3]
$P p_i = 1; pmtn; r_i \sum w_i T_i$	Baptiste (2002) [2]

- **maximal pseudopolynomially solvable:**

$Pm pmtn \sum w_i C_i$	McNaughton (1959) [27], Lawler et al. (1989) [22]
$Qm pmtn \sum w_i U_i$	Lawler (1979) [18], Lawler & Martel (1989) [23]

- **minimal NP-hard:**

* $P intree; pmtn; r_i C_{max}$	Lenstra (-) [24]
* $P p_i = 1; prec; pmtn C_{max}$	Ullman (1976) [31]
* $R2 chains; pmtn C_{max}$	Lenstra (-) [24]
* $P outtree; pmtn L_{max}$	Lenstra (-) [24]
$P2 pmtn; r_i \sum C_i$	Du et al. (1990) [11]
* $P pmtn; r_i \sum C_i$	Brucker & Kravchenko (2004) [7]
* $P2 chains; pmtn \sum C_i$	Du et al. (1991) [12]
* $R pmtn \sum C_i$	Sitters (2001) [29]
$P2 pmtn \sum w_i C_i$	Bruno et al. (1974) [8]
* $P p_i = p; pmtn; r_i \sum w_i C_i$	Leung & Young (1990A) [25]
* $P pmtn \sum w_i C_i$	Lenstra (-) [24]
* $P2 p_i = 1; chains; pmtn \sum w_i C_i$	Timkovsky (2003) [30], Du et al. (1991) [12]
* $P2 pmtn; r_i \sum w_i C_i$	Labetoulle et al. (1984) [17]
$P pmtn \sum U_i$	Lawler (1983) [20]
$P2 pmtn; r_i \sum U_i$	Du et al. (1992) [10]
* $P2 p_i = 1; chains; pmtn \sum U_i$	Baptiste et al. (2004) [3]
* $R pmtn \sum U_i$	Sitters (2001) [29]
$P p_i = p; pmtn \sum w_i U_i$	Brucker & Kravchenko (1999) [6]
$P2 pmtn \sum w_i U_i$	Single-machine problem

- **minimal open:**

$Pm p_i = 1; intree; pmtn; r_i C_{max}$	$P2 p_i = 1; intree; pmtn; r_i \sum C_i$	$P2 p_i = p; pmtn; r_i \sum U_i$
$Pm p_i = 1; prec; pmtn C_{max}$	$Pm p_i = p; chains; pmtn; r_i \sum C_i$	$R2 pmtn \sum U_i$
$Qm p_i = p; chains; pmtn; r_i C_{max}$	$Pm p_i = 1; intree; pmtn \sum C_i$	$Q2 p_i = p; pmtn \sum w_i U_i$
$Qm p_i = p; intree; pmtn C_{max}$	$Q2 p_i = p; chains; pmtn \sum C_i$	$P2 p_i = 1; chains; pmtn \sum T_i$
$Qm p_i = p; outtree; pmtn C_{max}$	$R2 pmtn \sum C_i$	$P2 p_i = p; pmtn; r_i \sum T_i$
$Pm p_i = 1; chains; pmtn; r_i L_{max}$	$P2 p_i = p; pmtn; r_i \sum w_i C_i$	$P2 pmtn \sum T_i$
$Pm p_i = 1; outtree; pmtn L_{max}$	$Q2 p_i = p; pmtn \sum w_i C_i$	$Q2 p_i = p; pmtn \sum T_i$
$Qm p_i = p; chains; pmtn L_{max}$		$P2 p_i = p; pmtn \sum w_i T_i$

- **maximal open:**

$Q outtree; pmtn; r_i C_{max}$	$Q p_i = p; prec; pmtn; r_i \sum C_i$	$Q pmtn \sum T_i$
$Q tree; pmtn C_{max}$	$Q p_i = p; pmtn; r_i \sum U_i$	$Qm p_i = p; prec; pmtn; r_i \sum T_i$
$Q chains; pmtn; r_i L_{max}$	$Rm pmtn \sum U_i$	$Rm pmtn \sum T_i$
$Q intree; pmtn L_{max}$	$Qm p_i = p; pmtn; r_i \sum w_i U_i$	$Q p_i = p; pmtn \sum w_i T_i$
$Qm prec; pmtn; r_i L_{max}$	$Q p_i = p; tree; pmtn; r_i \sum T_i$	$Qm p_i = p; pmtn; r_i \sum w_i T_i$

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